Sinho Chewi and Vrettos Moulos

HW 1

Due: June 22, 2018 at 10 PM

Sundry

Before you start your homework, write down your team. Who else did you work with on this homework? List names and email addresses. (In case of homework party, you can also just describe the group.) How did you work on this homework? Working in groups of 3-5 will earn credit for your "Sundry" grade.

Please copy the following statement and sign next to it:

I certify that all solutions are entirely in my words and that I have not looked at another student's solutions. I have credited all external sources in this write up.

1 Always True or Always False?

Classify the following statements as being one of the following and justify your answers.

- True for all combinations of x and y (Tautology)
- False for all combinations of x and y (Contradiction)
- Neither

(a) $x \wedge (x \implies y) \wedge (\neg y)$

(b) $x \implies (x \vee y)$

(c) $(x \vee y) \vee (x \vee \neg y)$

(d) $(x \implies y) \vee (x \implies \neg y)$

(e) $(x \vee y) \wedge (\neg(x \wedge y))$

(f) $(x \implies y) \wedge (\neg x \implies y) \wedge (\neg y)$

2 Propositional Practice

Convert the following English sentences into propositional logic and the following propositions into English. State whether or not each statement is true with brief justification.

(a) There is a real number which is not rational.

(b) All integers are natural numbers or are negative, but not both.

(c) If a natural number is divisible by 6, it is divisible by 2 or it is divisible by 3.

(d) $(\forall x \in \mathbb{R})\ (x \in \mathbb{C})$

(e) $(\forall x \in \mathbb{Z})\ ((2 \mid x \vee 3 \mid x) \implies 6 \mid x)$

(f) $(\forall x \in \mathbb{N})\ ((x > 7) \implies ((\exists a, b \in \mathbb{N})\ (a + b = x)))$

3 Prove or Disprove

(a) $\forall n \in \mathbb{N}$, if n is odd then $n^2 + 2n$ is odd.

(b) $\forall x, y \in \mathbb{R}$, $\min(x, y) = (x + y - |x - y|)/2$.

(c) $\forall a, b \in \mathbb{R}$ if $a + b \leq 10$ then $a \leq 7$ or $b \leq 3$.

(d) $\forall r \in \mathbb{R}$, if r is irrational then $r + 1$ is irrational.

(e) $\forall n \in \mathbb{N}^+$, $10n^2 > n!$.

4 Preserving Set Operations

For a function f, define the image of a set X to be the set $f(X) = \{y \mid y = f(x) \text{ for some } x \in X\}$. Define the inverse image of a set Y to be the set $f^{-1}(Y) = \{x \mid f(x) \in Y\}$. Prove the following statements, in which A and B are sets. By doing so, you will show that inverse images preserve set operations, but images typically do not.

Hint: For sets X and Y, $X = Y$ if and only if $X \subseteq Y$ and $Y \subseteq X$. To prove that $X \subseteq Y$, it is sufficient to show that $\forall x,\ x \in X \implies x \in Y$.

1. $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$.
2. $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$.
3. $f^{-1}(A \setminus B) = f^{-1}(A) \setminus f^{-1}(B)$.
4. $f(A \cup B) = f(A) \cup f(B)$.
5. $f(A \cap B) \subseteq f(A) \cap f(B)$, and give an example where equality does not hold.
6. $f(A \setminus B) \supseteq f(A) \setminus f(B)$, and give an example where equality does not hold.