CS 70 Discrete Mathematics and Probability Theory
Summer 2018 Sinho Chewi and Vrettos Moulos **DIS 6C**

1 Continuous Computations

Let X be a continuous random variable whose pdf is cx^3 (for some constant c) in the range $0 \le x \le 1$, and is 0 outside this range.

(a) Find c.

(b) Find $\mathbb{P}[1/3 \le X \le 2/3 \mid X \le 1/2]$.

(c) Find $\mathbb{E}(X)$.

(d) Find $\text{var}(X)$.

2 Arrows

You and your friend are competing in an archery competition. You are a more skilled archer than he is, and the distances of your arrows to the center of the bullseye are i.i.d. Uniform$[0,1]$ whereas his are i.i.d. Uniform$[0,2]$. To even out the playing field, you both agree that you will shoot one arrow and he will shoot two. The arrow closest to the center of the bullseye wins the competition. What is the probability that you will win? *Note*: The distances *from the center of the bullseye* are uniform.

3 Exponential Median

(a) Prove that if $X_1, X_2, \ldots, X_n$ are mutually independent exponential random variables with parameters $\lambda_1, \lambda_2, \ldots, \lambda_n$, then $\min(X_1, X_2, \ldots, X_n)$ is exponentially distributed with parameter $\sum_{i=1}^{n} \lambda_i$.
Hint: Recall that the CDF of an exponential random variable with parameter λ is $1 - e^{-\lambda t}$.

(b) What is the expected value of the median of three i.i.d. exponential variables with parameter λ?