

1 The Memoryless Property Uniquely Characterizes the Geometric Distribution

Let X be a discrete random variable which takes on values on \mathbb{Z}_+ . Suppose that for all $m, n \in \mathbb{N}$, we have $\mathbb{P}(X > m+n \mid X > n) = \mathbb{P}(X > m)$. Prove that X has the geometric distribution.

Hint: In order to prove that X is geometric, it suffices to prove that there exists a $p \in [0, 1]$ such that $\mathbb{P}(X > i) = (1 - p)^i$ for all $i > 0$.

2 Dice Games

Suppose you roll a fair six-sided die. You read off the number showing on the die, then flip that many fair coins.

(a) If the result of your die roll is i , what is the expected number of heads you see?

(b) What is the expected number of heads you see?

3 Pairwise Coupons

Vrettos and Sinho are each trying to collect a full set of n coupons. Every week, they each receive a randomly selected coupon in the mail. However, Vrettos will have a tantrum if Sinho gets a coupon he doesn't, so they agree that if they receive different coupons, they'll both just throw out the coupon they got. What is the expected number of weeks until they get a full set of coupons?