CS 70Discrete Mathematics and Probability TheorySummer 2018Sinho Chewi and Vrettos MoulosDIS 6A

1 Correlation and Independence

- (a) What does it mean for two random variables to be uncorrelated?
- (b) What does it mean for two random variables to be independent?
- (c) Are all uncorrelated variables independent? Are all independent variables uncorrelated? If your answer is yes, justify your answer; if your answer is no, give a counterexample.

2 Covariance

We have a bag of 5 red and 5 blue balls. We take two balls from the bag without replacement. Let X_1 and X_2 be indicator random variables for the first and second ball being red. What is $cov(X_1, X_2)$? Recall that $cov(X_1, X_2) = \mathbb{E}[XY] - \mathbb{E}[X]\mathbb{E}[Y]$.

3 Tale Sum

You're listening to a local storyteller reciting a tall tale. The storyteller will vary the length of the tale based on how excitedly the crowd seems to be listening; you've determined that for all t > 0, the probability that the story goes on for exactly t minutes is $\frac{t}{(t+1)!}$. What is the expected length of the tale?

Hint: You can write $\frac{t}{(t+1)!}$ as $\frac{t+1}{(t+1)!} - \frac{1}{(t+1)!}$.

Hint 2: Recall the power series for e^x : $e^x = \sum_{i=0}^{\infty} \frac{x^i}{i!}$.