1 Variance

This problem will give you practice using the "standard method" to compute the variance of a sum of random variables that are not pairwise independent. Recall that $\text{var}(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$.

(a) A building has n floors numbered $1, 2, \ldots, n$, plus a ground floor G. At the ground floor, m people get on the elevator together, and each person gets off at one of the n floors uniformly at random (independently of everybody else). What is the *variance* of the number of floors the elevator *does not* stop at? (In fact, the variance of the number of floors the elevator *does* stop at must be the same, but the former is a little easier to compute.)

(b) A group of three friends has n books they would all like to read. Each friend (independently of the other two) picks a random permutation of the books and reads them in that order, one book per week (for n consecutive weeks). Let X be the number of weeks in which all three friends are reading the same book. Compute $\text{var}(X)$.

2 Will I Get My Package?

A delivery guy in some company is out delivering n packages to n customers, where $n \in \mathbb{N}$, $n > 1$. Not only does he hand a random package to each customer, he opens the package before delivering it with probability $1/2$. Let X be the number of customers who receive their own packages unopened.

(a) Compute the expectation $\mathbb{E}(X)$.

(b) Compute the variance $\text{var}(X)$.

3 Binomial Conditioning

Let $n \in \mathbb{Z}_+$ and $p, q \in [0,1]$. Let $X \sim \text{Binomial}(n, p)$ and suppose that conditioned on $X = x$, $Y \sim \text{Binomial}(x, q)$. What is the unconditional distribution of Y?