CS 70 Discrete Mathematics and Probability Theory
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1 Anagrams

An anagram of a word is any re-ordering of the letters of the word, in any order. It does not have to be an English word.

(a) How many different anagrams are there of COVERAGE?

(b) How many different anagrams are there of COVFEFE?

(c) How many different anagrams are there of COVFEFES that contain EECS?

2 Counting Mappings

(a) A mapping $f : X \to Y$ is a function from X to Y, which assigns a unique element $f(x) \in Y$ for each $x \in X$. How many unique mappings are there between $X = \{1,2,\dots,n\}$ and $Y = \{1,2,\dots,m\}$?

(b) A mapping f is *injective* if for all $x_1, x_2 \in X$, $x_1 \neq x_2 \implies f(x_1) \neq f(x_2)$. How many injective mappings are there between $X = \{1,2,\dots,n\}$ and $Y = \{1,2,\dots,m\}$?

(c) Now suppose that we allow a mapping to be multi-valued, i.e. for each x, $f(x)$ is a subset of Y. How many unique multi-valued mappings are there between $X = \{1,2,\dots,n\}$ and $Y = \{1,2,\dots,m\}$?

3 Clothes and Stuff

(a) Say we've decided to do the whole capsule wardrobe thing and we now have only 5 different items of clothing that we wear (jeans, tees, shoes, jackets, and floppy hats, etc.). We have 3 variations on each of the items, and we wear one of each item every day. How many different outfits can we make?

(b) It turns out 3 floppy hats really isn't enough of a selection, so we've bought 11 more, and we now have 14 floppy hats. Now how many outfits can we make?

(c) If we own k different items of clothing, with n_1 varitions of the first item, n_2 variations of the second, n_3 of the third, and so on, how many outfits can we make?

(d) We love our floppy hats so much that we've decided to also use them as wall art, so we're picking 4 of our 14 hats to hang in a row on the wall. How many such arrangements could we make? (Order matters.)

(e) Ok, now we're packing for vacation to Iceland, and we only have space for 4 of our 14 floppy hats. How many sets of 4 could we bring? (Yeah, yeah, we knew you were going to use that notation. Now tell us the number as a function of d, your answer from the previous part.)

(f) Ok, turns out the check-in person for our flight to Iceland is being *very* unreasonable about the luggage weight restrictions, and we're going to have to leave some hats behind. Despite our best intentions, and having packed only 4 hats, we actually bought 18 additional floppy hats at the airport (6 in burgundy, 6 in forest green, and 6 in classic black). We'll keep our 4 hats that we brought from home, but we'll have to return all but 6 of the airport hats. How many color configurations can there be for the 6 airport hats that we keep?

4 Combinatorial Proof IX

Prove that for $0 < n < k$, $\binom{n}{k} = \sum_{i=0}^{k} \binom{n-i-1}{k-i}$.