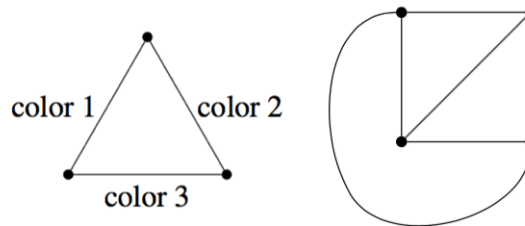


## 1 Edge Colorings

An edge coloring of a graph is an assignment of colors to edges in a graph where any two edges incident to the same vertex have different colors. An example is shown on the left.



- (a) Show that the 4 vertex complete graph above can be 3 edge colored. (Use the numbers 1, 2, 3 for colors. A figure is shown on the right.)
- (b) How many colors are required to edge color a 3-dimensional hypercube?
- (c) Prove that any graph with maximum degree  $d$  can be edge colored with  $2d - 1$  colors.
- (d) Show that a tree can be edge colored with  $d$  colors where  $d$  is the maximum degree of any vertex.

## 2 Bipartite Graph

A bipartite graph consists of 2 disjoint sets of vertices, such that no 2 vertices in the same set have an edge between them. Consider an undirected bipartite graph with two disjoint sets  $L, R$ . Prove that a graph is bipartite if and only if it has no tours of odd length.

## 3 Modular Arithmetic Equations

Solve the following equations for  $x$  and  $y$  modulo the indicated modulus, or show that no solution exists. Show your work.

(a)  $9x \equiv 1 \pmod{11}$ .

(b)  $10x + 23 \equiv 3 \pmod{31}$ .

(c)  $3x + 15 \equiv 4 \pmod{21}$ .

(d) The system of simultaneous equations  $3x + 2y \equiv 0 \pmod{7}$  and  $2x + y \equiv 4 \pmod{7}$ .