

1 Propositional Logic Language

For each of the following sentences, use the notation introduced in class to convert the sentence into propositional logic. Then write the statement's negation in propositional logic.

- (a) The cube of a negative integer is negative.
- (b) There are no integer solutions to the equation $x^2 - y^2 = 10$.
- (c) There is one and only one real solution to the equation $x^3 + x + 1 = 0$.
- (d) For any two distinct real numbers, we can find a rational number in between them.

2 Implication

Which of the following implications are always true, regardless of P ? Give a counterexample for each false assertion (i.e. come up with a statement $P(x, y)$ that would make the implication false).

- (a) $\forall x, \forall y, P(x, y) \implies \forall y, \forall x, P(x, y)$.
- (b) $\exists x, \exists y, P(x, y) \implies \exists y, \exists x, P(x, y)$.
- (c) $\forall x, \exists y, P(x, y) \implies \exists y, \forall x, P(x, y)$.
- (d) $\exists x, \forall y, P(x, y) \implies \forall y, \exists x, P(x, y)$.

3 Lewis Carroll

Here is an extract from Lewis Carroll's treatise *Symbolic Logic* of 1896:

(I) No one, who is going to a party, ever fails to brush his or her hair.

(II) No one looks fascinating, if he or she is untidy.

(III) Opium-eaters have no self-command.

(IV) Everyone who has brushed his or her hair looks fascinating.

(V) No one wears kid gloves, unless he or she is going to a party.

(VI) A person is always untidy if he or she has no self-command.

(a) Write each of the above six sentences as a quantified proposition over the universe of all people.

You should use the following symbols for the various elementary propositions: $P(x)$ for “ x goes to a party”, $B(x)$ for “ x has brushed his or her hair”, $F(x)$ for “ x looks fascinating”, $U(x)$ for “ x is untidy”, $O(x)$ for “ x is an opium-eater”, $N(x)$ for “ x has no self-command”, and $K(x)$ for “ x wears kid gloves”.

(b) Now rewrite each proposition equivalently using the contrapositive.

(c) You now have twelve propositions in total. What can you conclude from them about a person who wears kid gloves? Explain clearly the implications you used to arrive at your conclusion.